

PART - A

## Answer ALL questions

(10 $\times 2=20$ )
01 . What is a non-holonomic constraint? Give one example.
02. Prove that $\mathbf{F} . \mathbf{v}=\mathrm{dT} / \mathrm{dt}$ where T is the kinetic energy of the particle.

03 . Give an example of a velocity dependent potential.
04 . What is meant by principal moment of inertia and product of inertia?
05 . What are Euler's angles?
06. Show that the Hamiltonian is constant of motion if it is not an explicit function of time.
07. Show that the generating function $\mathrm{F}_{3}=\mathrm{pQ}$ generates an identity transformation with a negative sign.
08. Show that $\left[p_{y}, L_{z}\right]=p_{x}$
09. What is Jacobi identity?
10. Define Hamilton's principal function $S$

## PART - B

Answer any FOUR questions
$(4 \times 7.5=30)$
11. Set up the Lagrangian for a particle of mass $m$ in a central force using polar coordinates $(r, \theta)$ and hence obtain the differential equation of orbit of the form: $d^{2} u / d \theta^{2}+u=-m / l^{2} d / d u V(1 / u)$
12. Derive an expression for the Coriolis force and give one example as an illustration of the Coriolis force.
13. Illustrate the Lagrange's and the Hamiltonian methods of obtaining the equation of motion of a compound pendulum.
14. Given that the generating function for a harmonic oscillator is $F_{1}=(\mathrm{m} / 2) \omega q^{2} \cot \mathrm{Q}$. Show that the Hamiltonian of the oscillator transform to $K=\omega P$ and hence find $q(t)$.
15. Explain action and angle variables. Using the action- angle variable method show that the frequency of the one dimensional oscillator is $v=(1 / 2 \pi) \sqrt{ }(\mathrm{k} / \mathrm{m})$

## Answer any FOUR questions

16. a) Obtain Hamilton's canonical equations of motion from fundamental principles.
b) Consider a particle of mass $m$ moving in a plane under a central force $F(r)=-k / r^{2}+\mathrm{k}^{\prime} / \mathrm{r}^{3}$. Write the Lagrangian in terms of polar coordinates and find the equation of the orbit. $\quad(6.5+6)$
17. a) Obtain Euler's equations of motion for a rigid body acted upon by a torque $\mathbf{N}$.
b) If a rectangular parallelopiped with edges $2 \mathrm{a}, 2 \mathrm{a}, 2 \mathrm{~b}$ rotates about its centre of gravity under no forces, prove that its angular velocity about one principal axis is constant n and about the other axis is periodic $\mathrm{T}=2 \pi\left(\mathrm{a}^{2}+\mathrm{b}^{2}\right) /\left(\mathrm{b}^{2}-\mathrm{a}^{2}\right) . \boldsymbol{n}$
18. a) Obtain the transformation equations for the generating functions $\mathrm{F}_{1}(\mathrm{q}, \mathrm{Q}, \mathrm{t})$ and $\mathrm{F}_{4}(\mathrm{p}, \mathrm{P}, \mathrm{t})$
b) Show that the transformations given by $\mathrm{Q}=\mathrm{p}+\mathrm{iaq}$ and $\mathrm{P}=(\mathrm{p}-\mathrm{iaq}) / 2 \mathrm{ia}$ are canonical $\quad(8+4.5)$
19. a)Solve by the Hamilton-Jacobi method the motion of a particle in a plane under the action of a central potential $\mathrm{V}(\mathrm{r})$ to obtain the equation of orbit.
b) Solve the motion of a particle in one dimension whose Hamiltonian is given by $H=\mathrm{p}^{2} / 2 \mathrm{~m}+\mathrm{V}(\mathrm{q})$.
20. Write notes on any TWO of the following
i) Lagrange's equation from Hamilton's principle.
ii) Linear triatomic molecule.
iii) Solution to one dimensional harmonic oscillator by H-J method.
